Present Value

Lecture Slides 2

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Review from Lecture 1

- Objective: solid theoretical concepts $\rightarrow$ complex real world:
  - Lectures, exercizes + online trading.
- Finance and this course: deciding among different investment alternatives:
  - Relative Valuation.
- The LOP, PPP and Arbitrage + opportunity cost.
- Projects and firms.
- Bonds or debt and stocks or equity.
- Firms vs. individuals, the perfect market and ethical dilemmas.

In Lecture II

Assuming the simplest world possible

- The perfect market assumptions.
- The main -and most basic- valuation tools and concepts:
  - Rate of Return (and interest rates).
    - Definition.
    - Compounding/ calculating.
  - The Time Value of Money.
  - Present Value and Net Present Value.

The concept of Rate of Return

The cornerstone of finance

- **Rate of return** $\Longrightarrow$ money today more valuable than money tomorrow.
  `http://www.investopedia.com/terms/r/rateofreturn.asp`
- Two sides of the market:
  - Investors: maximize cash flows.
  - Firms: capital budgeting.
- $\exists$ a best method to take investment and firms' decisions: **NPV**.

Perfect market: the four basic asumptions

- No taxes.
- No transaction costs.
- No differences in information or opinions among investors.
- "Infinite" agents both in **demand** and **supply**.

The basic scenario: the perfect market

Why?!?

simplicity

Make everything as simple as possible, but **not simpler**!!!

Other simplifying assumptions

Not part of the perfect market assumptions

- Interest rate is the same in every period.
- No **inflation**.
- No risk or uncertainty: perfect foresight.

Loans and Bonds I

Definitions

- **Loan** definition. **Maturity**. **Interest**.
- **Bond** $\leftrightarrow$ binds. Fixed income. Corporate project.
- Buying a bond as extending a loan. Investor.
- Giving, issuing or selling a bond. Firm.

Loans and Bonds II

Interest rate

- Interest payment vs. nointerest payment.
- Rate of return vs. interest rate.
- Buying a bond vs. putting the money into a bank savings account (flexible).

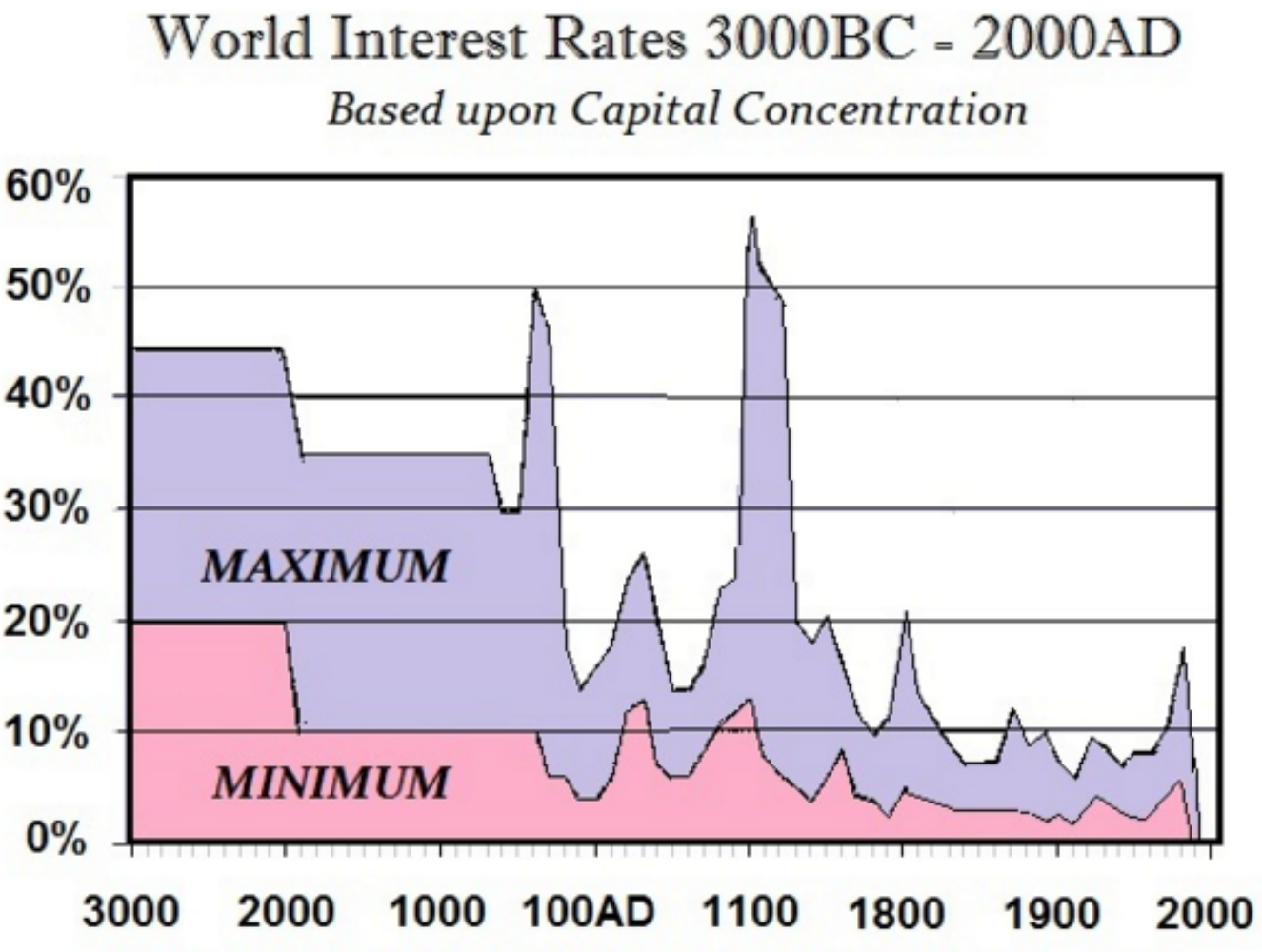
Returns

Returns, Net Returns, and Rates of Return

- Return: $C_1 = Return\ at\ Time\ 1\ =\ \$15$. Purchase price: $C_0 = \$10$.
- Net return:
  - $from\ Time\ 0\ to\ Time\ 1 =\ \$15\ -\ \$10\ =\ \$5$
  - $Net\ Return_{0,1}\ =\ C_1\ -\ C_0$.
- Rate of return:
  - $r_{0,1} = r_1 = \frac{Net\ Return\ from\ Time\ 0\ to\ Time\ 1}{Purchase\ Price\ at\ Time\ 0}$.
  - $r_1 = \frac{C_1 - C_0}{C_0} = \frac{\$15 - \$10}{\$10} = 0.5 = 50\%$.

Anecdote 2.2.a

Interest Rates over the Millennia



- Armstrong Economics, based on Homer and Sylla, "History of Interest Rates".

Interim payments

- Dividends (stocks).
- Coupons (bonds).
- Interim rent (real estate).

Calculating rate of return:

$$r_1 = \frac{C_1 + All\ dividends\ from\ 0\ to\ 1 - C_0}{C_0} = \underbrace{\frac{C_1 - C_0}{C_0}}_{Capital\ Gain,\ \text{in}\ \%} = \underbrace{\frac{All\ Interim\ Ys}{C_0}}_{Interim\ Yield}$$

$$r_1 = \frac{\$15 + \$5 + \$10}{\$10} = \underbrace{\frac{\$15 - \$10}{\$10}}_{Capital\ Gain,\ \text{in}\ \%} + \underbrace{\frac{\$5}{\$10}}_{Interim\ Yield} = 1 = 100\%$$

Common errors

- Call return to rate of return.
- Calling yield -what means rate of return- to interim payments.
- The interest rate has just increased by 5%... Basis points $\rightarrow$

Concept 2.2.1: basis points

Definition

A **basis point** is a $1/100$ of a percent. 100 basis points $= 1\%$

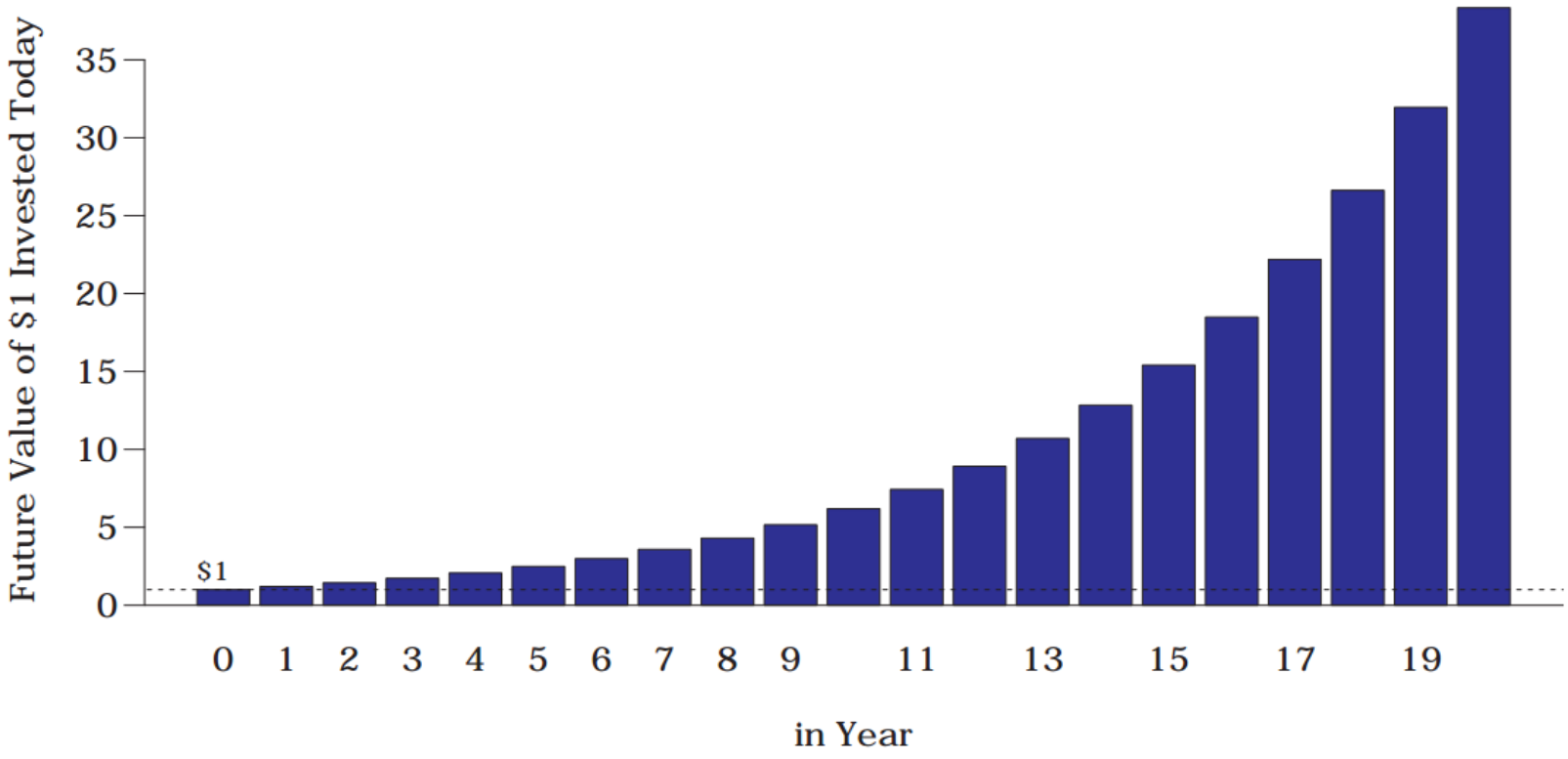
- Basis points (0.01%) and points (1%) help with "percentage ambiguities".

Questions 2.2.a

1. What are the four perfect market assumptions?
2. Is a deposit into a savings acount more like a long-term bond investment or a series of short-term investments?
3. An investment costs \$1,000 and pays a return of \$1,125. What is the rate of return?
4. An investment costs \$1,000 and pays a net return of \$75. What is its rate of return?
5. Is 5 the same as 500%?
6. By how many basis points does the interest rate changes if it increases from 9% to 12%?
7. If an interest rate of 10% decreases by 20 basis points, what is the new interest rate?
8. Does it logically make sense to have negative interest rates?

Time Value, Future Value, and Compounding

- Crucial quantitatively (compounding over 20 years at 20% per annum):



- Crucial in practice: "They cannot even estimate a compounded interest".

The Future Value of Money

- How much will you receive next year if $r = 10\%$ and you invest \$100 today?
- Turning around the r formula:

$$r_1 = \frac{C_1 - C_0}{C_0} \iff C_0 \cdot (1 + r_1) = C_1 \Rightarrow \$100 \cdot (1 + 0.1) = \$110$$

- The **time value of money** causes the future value (**FV**) \$110 to be higher than the present value (**PV**) \$100:

$$r_1 = \frac{FV - PV}{PV} \iff FV = PV \cdot (1 + r_1)$$

Compounding and Future Value

What if you could earn that 10% year after year?

You will have \$110 in year 1, reinvest:

$$C_2 = C_0 \cdot (1+r)^2 = \$100 \cdot (1+0.1)^2 = C_1 \cdot (1+r) = \$110 \cdot (1+10\%) = \$110 \cdot 1.1$$

- This \$121 is a FV of \$100 today, and represents a total 2-year r:

$$r_2 = \frac{C_2 - C_0}{C_0} = \frac{\$121 - \$100}{\$100} = 21\%$$

- Notice it´s more than 20% $(10\% + 10\%)$:
  - **Compound interest**: you **earn interest on interest**!!!
- The 3-year **holding rate of return**:

$\$100 \cdot (1+0.1)^3 = \$133.1.$ $r_3 = \frac{C_3 - C_0}{C_0} = 33.1\%$.

The Compounding Formula

The "one-plus formula"

$$\underbrace{(1+r_t)}_{\text{Multiperiod Holding Rate of Return}} = \underbrace{(1+r)^t}_{\text{Multiperiod Holding Rate of Return}} = \underbrace{(1+r)}_{\text{Current 1-Period Spot Rate of Return}} \cdot \underbrace{(1+r)}_{\text{Next 1-Period Rate of Return}} \cdots \underbrace{(1+r)}_{\text{Final 1-Period Rate of Return}}$$

- Easy to use: $C_t = Investment \times (1+r)^t - Investment$; $r_t = (1+r)^t - 1$.
- If you want to know what constant two 1-year r would give you a 50% rate of return, solve:

$$(1+r)^2 = 1.5 \iff r = \sqrt[t]{1+r_t} - 1 = \sqrt{1.5} - 1 \approx 22.47\%$$

Anecdote 2.2.b

Life Expectancy and Credit

- 30-year bonds existed when life expectancy was only 25.
- Hammurabi, 1700 B.C.E.
- How long it takes for 1 mina of silver, growing at 20% of interest per year, to reach 64 minae: Logs required!

How long before doubling or tripling?

$$(1+r)^t = (1+r_t) \iff t = \frac{log(1+r_t)}{log(1+r)}$$

$$(1+5\%)^x = (1+1) = (1+100\%) \iff x = \frac{log(1+100\%)}{log(1+5\%)} = \frac{log(2)}{log(1.05)} \approx 14.2$$

Risky mistake 2.2.a

Adding or Compounding Interest Rates?

X „,Thinking in arithmetic terms for wealth accumulation: high volatility funds look particularly good.

- Small periods, investments and interest rates: adding not huge error (20% vs. 21%).
- Large periods, large investments or large interest rates: huge errors:
- Be careful about how banks quote interest rates.

COMPOUND!!!

Questions 2.2.b

1. If the 1-year rate of return is 10% and interest rates are constant, what is the 10-year holding rate of return?
2. If you invest \$3,000 today and it earns 5% per year, how much will you have in 10 years?
3. What is the holding rate of return for a 15-years investment that earns 10%/year each year? What would a \$1,000 investment grow into?
4. A project lost one-fourth of its value each year for 3 years. What was its total holding rate of return? How much is left if the original investment was \$50,000?
5. If the 10-year holding rate of return is 80% and interest rates are constant, what is the (compounding) annual interest rate?

Present Value, Discounting, and Capital Budgeting

- If project X wil return \$1 million in 5 years, how much should you be willing to pay today? **Capital budgeting**.
- Again (PV):

$$C_0 = \frac{C_1}{1+r_1} = PV(C_1) = \frac{\$100}{1+0.1} = \frac{\$100}{1.1} \approx \$90.91$$

- Check the previous formula reversed: investing \$90.91at a $r = 10\%$.
- **Cost of capital** and **opportunity cost**.

The Cornerstones of Finance

Crucial formulas to UNDERSTAND and Learn

- Rate of Return: $r_t = \frac{C_t - C_0}{C_0}$
- Future Value: $FV_t = C_t = C_0 \cdot (1 + r_t) = C_0 \cdot (1 + r)^t$
- Compounding: $(1 + r_t) = (1 + r) \cdot (1 + r) \cdots (1 + r)$
- Present Value: $PV_t = C_0 = \frac{C_t}{(1+r_t)} = \frac{C_t}{(1+r)^t}$.
- Discount Factor: $\frac{1}{(1+r_t)} = \frac{1}{(1+r)^t}$

Questions 2.2.c

1. A project with a cost of capital of 25% pays off \$300. What should it cost today?
2. What are the units on rates of return, discount factors, future values, and present values?
3. Would it be good or bad for you, in terms of the present value of your liabilities, if your opportunity cost of capital increased?

Net Present Value I

Definition and formula

- All cash flows are translated into cash today:

1. Convert all cash flows into their present values.
2. Add them (subtracting costs).

- $NPV = C_0 + PV(C_1) + PV(C_2) + \cdots = C_0 + \frac{C_1}{1+r_1} + \frac{C_2}{1+r_2} + \cdots = C_0 + \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \cdots$

- NPV capital budgeting rule: accept if NPV>0, reject if NPV<0.

Net Present Value II

An example: a Hypothetical Project Cash Flow Table

Time	Project Cash Flow	Interest Rate: Annualized	Interest Rate: Discount Holding	Present Factor	Value
t	C_t	r	r_t	$\frac{1}{(1+r)^t}$	$PV(C_t)$
Today 0	–\$900	5.00%	0.00%	1.0000	–\$900.00
Year +1	+\$200	5.00%	5.00%	0.9524	+\$190.48
Year +2	+\$200	5.00%	10.25%	0.9070	+\$181.41
Year +3	+\$400	5.00%	15.76%	0.8638	+\$345.54
Year +4	+\$400	5.00%	21.55%	0.8227	+\$329.08
Year +5	–\$100	5.00%	27.63%	0.7835	–\$78.35
				Net Present Value (Sum):	\$68.15

- Do you reject or accept taking this project?

Questions 2.2.d

1. What is the NPV budgeting rule?
2. You are considering a 3-year lease for a building, where you have to make one payment now, one in a year, and a final one in two years:
    1. Would you rather pay \$200,000 up-front, then \$100,000 each in the following two years, or would you rather pay \$140,000 each year?
    2. If the interest rate is 5%, what equal payment amount (rather than 140,000%) would leave you indifferent? (The EAC).

Application 2.2.a

Are Faster-Growing Firms Better Bargains?

- Company A will produce over the next 3 years:

$$A_1 = \$200 \ ; \ A_2 = \$250 \ ; \ A_3 = \$300$$

- Company B will produce:

$$B_1 = \$200 \ ; \ B_2 = \$180 \ ; \ B_3 = \$150$$

Is it A a better company to buy than B?

Questions 2.2.e

- A month ago a firm suffered a large court award against it that will force it to pay compensatory damages of $200 million next January 1. Are shares in this firm a bad buy until January 2?

Summary

- The perfect market and the four basic assumptions.
- A bond. Buying a bond. Issuing a bond.
- Basis points.
- The time value of money.
- Averaging and compounding returns.
- Interest rate quotes. If doubt ask!
- Net present value.

Key Concepts I

The student completes the empty definitions

- Rate of return: The gain or loss on an investment over a specified period, expressed as a percentage increase over the initial investment cost.
- NPV:
- Demand:
- Supply:
- Inflation
- Loan:
- Maturity:

Key Concepts II

- Interest:
- Bond:
- Interim payment:
- Basis point:
- Time Value of Money:
- Future Value:
- Present Value:

Key Concepts III

- Compound interest:
- Holding rate of return:
- Capital budgeting:
- Cost of capital:
- Opportunity cost:

Assignments for next lecture I

- Complete the definition of the concepts above (those we have explained today).
- Do quiz 1 online.
- Keep trading online:
  - Take into account today's lessons.
  - Registry your progress.
- Read chapter 2 of the textbook (again).

Assignments for next lectures II

- Do quiz 2 online.
- Keep trading online and recording your progress.
  - Presentations 1-5: prepare!
- Read chapter 3 of the textbook and the lecture notes.
- Complete the rest of the concepts above.