Time-Varying Rates of Return and the Yield Curve

Chapter 5, slides 5.1

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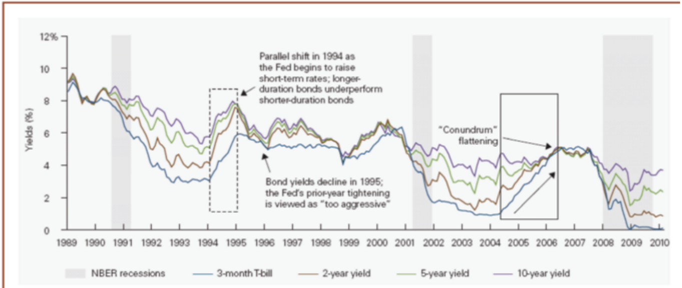
When Rates of Return are Different

- Time dependent or horizon dependent interest rates: over time and depending on maturity.

- Examples:
  - U.S Treasury bonds: 20-year, 2.30% per annum; 1-year, 0.56% (22nd April).

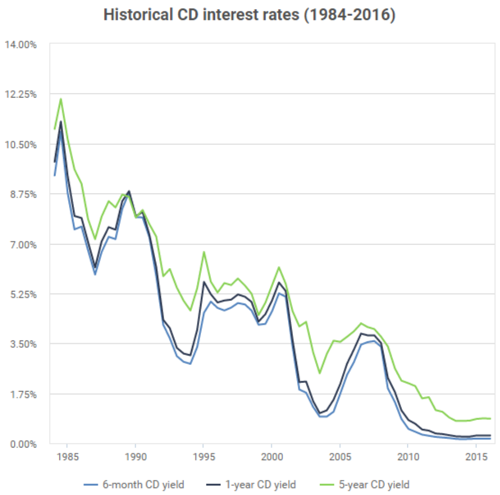
Figure 3. Various Constant-Maturity Treasury Yields Since January 1989



Sources: Federal Reserve and National Bureau of Economic Research.

Certificate of Deposit

Historical interest rates



Why do time-dependent interest rates matter?

Who cares?

- All investors and CEOs.
- Why? You need to compare:
  - Short-term and long-term projects.
  - Shor-term and long-term financing (costs).
- How?? Discounting, cost of capital $\leftrightarrow r_L$, r_S.

In this chapter: time-dependent r and inflation

- Working with time-varying rates of return.
- Inflation.
- Time-varying interest rates: U.S treasuries and the yield curve.
- Why is the (nominal) yield curve usually upward sloping?
- Corporate insights about time-varying costs of capital from the yield curve.

1. Working with Time-Varying Rates of Return

- Interest rates change over time and for bonds/investments with different maturities.
- We need to know how to work in this setting: here the tools.
- All the previous tools remain applicable. Changes? $\Rightarrow$

1.1. Compounding Different Rates of Return

- Compounding different rates of return:

$$(1 + r_{0,1}) \cdot (1 + r_{1,2}) \cdot (1 + r_{2,3}) \cdots (1 + r_{T-1,T}) = (1 + r_{0,T})$$

- Subscripts. Show example.
- Spot and forward rates.

Questions 5.1.a

1. If the first-year interest rate is 3% and the second year interest rate is 2%, what is the two-year total interest rate?
2. Althought a two-year project had returned 25% in its first year, overall it lost half of its value. What was the project's rate of return after the first year?
3. A project lost 1/4 of its value the first year, then gained 40% of its value, then lost 2/3 of its value, and finally tripled in value. What was the average rate of return? What was the investment's overall four-year rate of return? If one is positive, is the other, too?

Annualized Rates of Return

- Difficult to compare interest rates if they are not annualized. eg: 28% over 8.34 years $\Rightarrow$we annualized them.
- Average annualized rate of return (ARR): does not mean... as a car average speed!
- How to compute it?
  - If $r_3 = 75\%$, is ARR=0.25%?? No!! $(1.25)^3 \approx 1.95 \implies r_3 = 0.95 \neq 0.75$.
  - Right: $(1 + r_{\bar{3}}) \cdot (1 + r_{\bar{3}}) \cdot (1 + r_{\bar{3}}) = 1 + r_3; \implies (1 + r_{\bar{3}})^3 = (1 + 0.75)$ or in general $(1 + r_{\bar{t}})^t = (1 + r_{0,t})$
- Where $r_{\bar{t}}$ is the ARR and $r_{0,t}$ is the holding rate of return. Real world: quoted as ARR.

The total holding rate of return

Compounding it: Total Holding Rate of Return

IMPORTANT: The total holding rate of return over t years, called $r_{0,t}$, is translated into an annualized rate of return , called $r_{\bar{t}}$, by taking the tth root:

$$(1 + r_{\bar{t}}) = \sqrt[t]{1 + r_{0,t}} = (1 + r_{0,t})^{1/t}$$

Compounding the annualized rate of return over t years yields the total holding rate of return.

Questions 5.1.b

1. If you earn a rate of return of 3% over 4 months, what is the annualized rate of return?
2. Assume that the two-year holding rate of return is 30%. The average (arithmetic) rate of return is therefore 15%. What is the annualized (geometric) rate of return? Is the annualized rate the same as the average rate?
3. Is the compounded rate of return higher or lower than the sum of the individual rates of return? Is the annualized rate of return higher or lower than the average of the individual rates of return? Why?
4. If the total holding interest rate is 40% for a five-year investment, what is the annualized rate of return?
5. If the per-year interest rate is 8% for each of the next 5 years, what is the annualized five-year rate of return?

Present Values with Time-Varying Interest Rates

NPV: nothing new

$$
\begin{aligned}
\text{NPV} = \text{PV}(C_0) &+ \text{PV}(C_1) + \text{PV}(C_2) + \text{PV}(C_3) + \cdots \\
= C_0 &+ \frac{C_1}{1 + r_{0,1}} + \frac{C_2}{1 + r_{0,2}} + \frac{C_3}{1 + r_{0,3}} + \cdots \\
= C_0 &+ \frac{C_1}{1 + \bar{r}_1} + \frac{C_2}{(1 + \bar{r}_2)^2} + \frac{C_3}{(1 + \bar{r}_3)^3} + \cdots \\
= C_0 &+ \frac{C_1}{1 + r_{0,1}} + \frac{C_2}{(1 + r_{0,1}) \cdot (1 + r_{1,2})} + \frac{C_3}{(1 + r_{0,1}) \cdot (1 + r_{1,2}) \cdot (1 + r_{2,3})} + \cdots
\end{aligned}
$$

- Just be careful with subscripts.

A project's NPV

- Having an upward sloping structure of interest rates: $r_{\bar{t}}$ is 5% over one year, 0.5% more for every subsequent year:

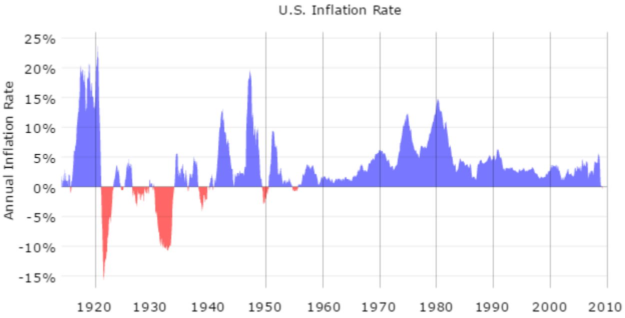
Time	Project Cash Flow	Interest Rate		Discount Factor	Present Value
		In Year	Compounded		
t	C_t	$r_{\bar{t}}$	$r_{0,t}$	$\frac{1}{1+r_{0,t}}$	$PV(C_t)$
Today	−\$900	any	0.0%	1.0000	−\$900.00
Year 1	+\$200	5.0%	5.0%	0.9524	\$190.48
Year 2	+\$200	5.5%	11.3%	0.8985	\$179.69
Year 3	+\$400	6.0%	19.1%	0.8396	\$335.85
Year 4	+\$400	6.5%	28.6%	0.7773	\$311.04
Year 5	−\$100	7.0%	40.3%	0.7130	−\$71.33
			Net Present Value (Sum):		\$45.73

Questions 5.1.c

- A project costs \$300 and will provide cash flows of $+\$150$, $+\$250$, and $+\$450$ in consecutive years. The annualized interest rate is 4% per annum over one year, 3.5% per annum over two years, and 4.5% per annum over three years. What is the project's NPV?

Inflation

- Inflation and deflation: videos.
- Inflation $\uparrow \rightarrow$ money value $\downarrow$, prices $\uparrow$. eg inflation 100%: an apple \$0.5 today $\Rightarrow$ \$1 next year.



- Does it matter for finance?? Real vs. nominal terms and contracts. In the US...

Measuring the Inflation Rate

Contracts in nominal terms: what effect does inflation have on returns?

- Prices change differently!!! $\Rightarrow$
  - Define baskets or bundles of representative goods and services.
  - Measure an average price change for these items.
  - BLS (US), and CPI: 40% housing, 20% food, 15% transportation... Check!
- Many contracts specifically indexed to a particular inflation definition. (There are many).

IMPORTANT: The common statement "in today's dollars" is ambiguous. Some people mean "inflation adjusted." Other people mean present values (i.e., "compared to an investment in risk-free bonds"). When in doubt, ask!

Real and nominal interest rates

How to index a contract for inflation

- Real interest rate: takes inflation out from the nominal rate $\Rightarrow$ calculate r as if there was not inflation.

- Time value of money, NPV: we care about real rates!!!

- Simple (exagerated) scenario: $\pi = 100\%$ per year, bond's $r_n = 700\%$, $r_r =??$

$$(1 + r_n) = (1 + r_r) \cdot (1 + \pi) \; or \; (1 + r_r) = \frac{(1 + r_{nominal})}{(1 + \pi)}$$

- Real world: r_n -almost- always > 0, r_r not so.

Questions 5.1.d

- The nominal interest rate is 5%, inflation is 2%. What is the real interest rate?

Inflation in Net Present Values

Simple rule: never mix apples and oranges

- Beauty of NPV: everything is translated into the same units, $ today. Keep it like this!!
  - Discount nominal cash flows with nominal interest rates.
  - Discount real (inflation-adjusted) cash flows with real interest rates.
  - Never discount nominal cash flows with real interest rates, or vice versa!

Example

- The real interest is 3% per annum and the inflation rate is 8% per annum. What is the present value of a \$500,000 nominal payment next year?

1. Compute the nominal rate and the real payment:
   1. r_n : we know
      $(1 + r_n) = (1 + r_r)(1 + \pi) \iff (1 + r_n) = 1.03 \cdot 1.08 = 1.1124$, so
      $r_n = 11.24\%$
   2. R, the real cash flow:
      $N = R \cdot (1 + \pi) \iff R = \frac{N}{1+\pi} \Rightarrow R = \frac{\$500{,}000}{1.08} \approx \$462,962.96$
2. Nominal terms PV: $PV = \frac{\$500{,}000}{1.1124} = \$449,478.60$.
3. Real terms PV: $PV = \frac{\$462{,}962.96}{1.03} = \$449,478.60$.

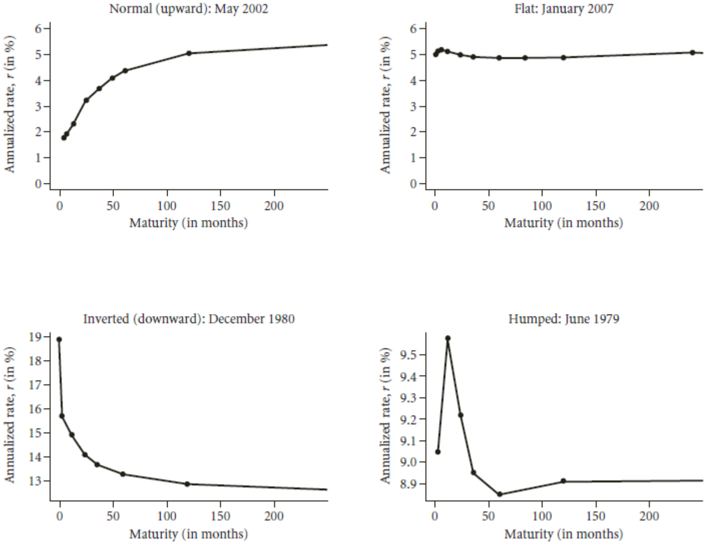
- Why are they equal? $PV = \frac{N}{1+r_n} = \frac{R}{1+r_r} = \frac{R \cdot (1+\pi)}{(1+r_r) \cdot (1+\pi)}$.

U.S Treasuries and the Yield Curve

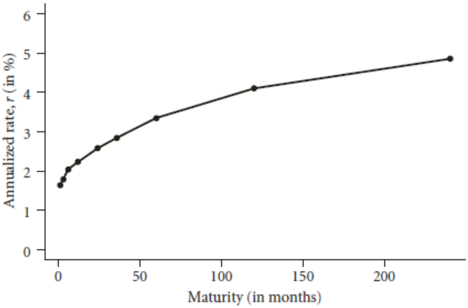
- A very important and simple market. Cannot fail to pay: promise $, and U.S. has the right to print them. 3 types of Treasuries:
  - Treasury bills (T-bills): maturities of less than 1 year.
  - Treasury notes: maturities between 1 and 10 years.
  - Treasury bonds: maturities greater than 10 years.
- After they are sold by the gvt. they are actively traded. Larger buyers and sellers.
  - Many buyers and sellers. Huge volume ($560 billion per trading day in 2006: about 10 times US GDP).
  - Very low transaction costs, and taxes.
  - Differences of opinion??
- At any point in time, interest rates on Treasuries vary, depending on... maturity: the yield curve.

The Yield Curve, Shapes

Video



An Example: The Yield Curve on December 31, 2004



	← Months			Years →						
	1	3	6	1	2	3	5	7	10	20
Dec. 31, 2004	1.63%	1.79%	2.05%	2.23%	2.58%	2.85%	3.35%	3.75%	4.10%	4.85%

- Why does it -frequently- slope upwards? Higher inflation expectations/uncertainty (little) + higher future -short term- interest rates (no) + bargains on the long end (no) + compensating investors for -interim- risk (yes). Explain, page 109.

Using the yield curve

- If you had purchased a 3-month Treasury on Dec. 31st, with an $r_{\overline{0,1/4}} = 1.63\%$, then a \$100 investment would turn into:

$$\$100 \cdot (1 + 1.63\%)^{1/4} \approx \$100 \cdot 1.0041$$

- Investing \$500,000 into 2-year notes, return by 31st Dec 2006:

$$r_{0,2} = (1 + r_{\bar{2}}) \cdot (1 + r_{\bar{2}}) - 1 \approx 1.0258 \cdot 1.0258 - 1 \approx 5.23\% \Rightarrow$$

$C_2 \approx (1 + 5.23\%) \cdot \$500,000 = \$526,150$.

- One inaccuracy: we are using treasuries as zero bonds, while they tend to pay bi-annually.

Forward rates of return

Understand them properly. I will not ask you to estimate them

- The happen in the future: $r_{1,2}$.
- Important to understand subscripts: $r_{0,t}$,$r_{\bar{t}}$,$r_{1,2}$: holding, annualized and forward.

TABLE 5.1 RELATION BETWEEN HOLDING RETURNS, ANNUALIZED RETURNS, AND YEAR-BY-YEAR RETURNS ON DECEMBER 31, 2004, BY FORMULA

	Rates of Return		
Maturity	Total Holding	Annualized	Compounded Rates
1 Year	$(1 + 2.23\%)$	$= (1 + 2.23\%)^1$	$= (1 + 2.23\%)$
	$(1 + r_{0,1})$	$= (1 + r_{\bar{1}})^1$	$= (1 + r_{0,1})$
2 Years	$(1 + 5.23\%)$	$\approx (1 + 2.58\%)^2$	$\approx (1 + 2.23\%) \cdot (1 + 2.93\%)$
	$(1 + r_{0,2})$	$= (1 + r_{\bar{2}})^2$	$= (1 + r_{0,1}) \cdot (1 + r_{1,2})$
3 Years	$(1 + 8.80\%)$	$\approx (1 + 2.85\%)^3$	$\approx (1 + 2.23\%) \cdot (1 + 2.93\%) \cdot (1 + 3.39\%)$
	$(1 + r_{0,3})$	$= (1 + r_{\bar{3}})^3$	$= (1 + r_{0,1}) \cdot (1 + r_{1,2}) \cdot (1 + r_{2,3})$

Bond Payoffs and Your Investment Horizon

If Treasuries offer different annualized r_s over different horizons, do corporate projects have to do so?

- Yes! they compete with Treasury bonds for investors' money.
- If you buy a 3-year bond and sell it after 1 year, the r you will receive is that of 1 year bond.

Questions 5.1.e

- A ten-year and a one-year zero-bond both offer an interest rate of 5% per annum:

1. How does an increase of 10 basis points in the prevailing interest rate change the value of the one-year bond? What about the value of the ten-year bond?
2. What is the ratio of the value change over the interest change?

Questions 5.1.f

- On May 31, 2002, the Wall Street Journal reported that a thirty-year inflation-adjusted bond offered a real yield of about 3.375% per year. The current inflation rate was only 1.6% per year, and a normal thirty-year Treasury bond (not inflation adjusted) offered a nominal yield of 5.600% per year. In what inflation scenario would you have been better off buying one or the other?

Corporate Time-Varying Costs of Capital

- Now: you understand why the yield curve is upward sloping.
- Resemblance with corporate bonds!!! $\Longrightarrow$ longer term projects -in general- higher rates of return.
- Higher expected rate of return $\neq$ higher NPV (in a world without perfect foresight).

> - The appropriate cost of capital (rate of return) should usually depend on how long term the project is.
> - Short-term corporate projects usually have lower costs of capital than long-term projects.
> - Conversely, corporations usually face lower costs of capital (expected rates of return offered to creditors) if they borrow short term rather than long term.

Keywords

Annualized rate, average rate of return, Bureau of Labor Statistics, CPI bond, CPI, Forward rate, GDP Deflator, Inflation, Hyperinflation, Long bond, nominal return, nominal terms, PPI, paper loss, real return, nominal terms, real terms, reinvestment rate, spot rate, T-bill, TIPS, term premium, Treasuries, Treasury bill, Treasury bond, Treasury note, yield curve.